

The Impact of Competition and Time-to-Finance on Corporate Cash Holdings*

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The paper contains graphs in colors

Abstract

In this paper we introduce time frictions in the capital market to show how industry specific competition explains the variation in corporate cash holdings. We show that time-to-finance is positively related to cash when firms are exposed to preemption risk, and that the role of cash changes with the level of preemption risk. Firms facing a high risk of preemption put a higher value on cash for investments and a lower value on cash held for hedging illiquidity risk. Additionally, cash holdings are hump-shaped in competition in the presence of time-to-finance, which reconciles some of the earlier mixed empirical findings.

JEL classification: G30, G32, G35

Keywords: cash holdings; competition; real investment; capital market frictions

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1 Introduction

How can we explain the substantial variation in corporate cash holdings? This question has recently received increasing attention in the literature (e.g. Almeida et al., 2004; Bates et al., 2009; Brown and Petersen, 2011; Lins et al., 2010; Qiu and Wan, 2015). However, we are still far from having a complete explanation of corporate cash behavior. In this paper we show how financial frictions affect firms in different ways depending on industry specific characteristics of competition. More specifically time frictions in capital markets have a larger impact on corporate cash holdings when firms compete for investment opportunities which are mutually exclusive, short-lived, and associated with a high winner's advantage. This is typically true for industries in which firms compete for patents and acquisitions. Our model allows us to reconcile some of the mixed empirical evidence on the relation between competition and corporate cash holdings (see e.g Grullon and Michaely, 2007; Lyandres and Palazzo, 2014; Morellec et al., 2014) and helps explain the observed differences in cash holdings between industries.

We develop a dynamic model of investment, financing and cash management decisions with uncertain and lumpy investment opportunities, in which firms are exposed to competition and time frictions in the capital market¹. We focus on the competition in patenting and acquisition markets which is modeled as the threat of losing a growth option due to preemption. We refer to this as the firm's exposure to preemption risk. The model provides a series of empirical testable predictions. Firstly, time-to-finance increases corporate cash holdings. Hence, firms with less access to capital markets should hold more cash. Secondly, firms subject to higher levels of preemption risk hold more cash on their balance sheet. Finally, we predict a hump-shaped relation between preemption risk and corporate cash holdings in the presence of time-to-finance.

To understand why firms hold substantial amounts of cash on their balance sheet, one needs to understand the financial frictions present in capital markets. In a frictionless world, firms would be able to obtain an unlimited amount of financing at any point in time without incurring

¹Through out the paper we use term *time-to-finance*, introduced by Ma et al. (2014), to denote the time associated with obtaining external financing due to search frictions in capital markets and capital supply uncertainty.

any costs. However, in the presence of financial frictions, firms may hedge themselves against liquidity shortfalls by carrying cash buffers within the firm. Hence, financial frictions creates a precautionary motive for cash as described by Keynes (1936). The most dominant explanation of corporate cash holdings is the costs associated external financing due to asymmetric information (Myers and Majluf, 1984). Hennessy and Whited (2007) show that the estimated marginal equity flotation costs range from 5.0% to 10.7% for small and large firms respectively. Several papers have shown how the presence of such external financing costs lead firms to optimally retain cash. This is done to either hedge against illiquidity induced default (Duchin, 2010; Gryglewicz, 2011; Anderson and Carverhill, 2012; Harford et al., 2013; Bolton et al., 2014; Hugonnier et al., 2015) or to enable themselves to fund future profitable investment opportunities internally (Almeida et al., 2004; Acharya et al., 2007; Kisser, 2013). We add to the discussion of the role of corporate cash, by showing that the purpose of cash depends on the industry specific characteristics of competition.

There are several channels through which competition may affect corporate cash holdings. Competition may reduce expected earnings, increase earnings volatility or lead to lost investment opportunities (see e.g. Mason and Weeds, 2010; Lyandres and Palazzo, 2014; Morellec et al., 2014). It is therefore important to specify what is meant by a high level of competition, as the way firms compete are likely to vary with the industry in which they operate. In our model we focus on industries in which firms primarily compete for patents and the acquisition of small high growth potential firms. These can be characterized as industries where investment opportunities are mutually exclusive, i.e. there is a high level of preemption risk. Moreover, investments opportunities are often associated with a high winner's advantage and a significant loss in market share of the losing firm. Hence, very valuable growth options may loose their value, either due to technological obsolescence or simply the loss of a potential acquisition due to the investment of a competitor. This reduces the life time of the firm's investment opportunities, forcing it to invest sooner to ensure continuing growth².

²One specific example of this is the mobile phone market. In the early 2000's Nokia had just overtaken Motorola to become the most dominant player within the mobile phones sector with a market share of around 31% (Barwise and Meehan, 2011). However, as Apple introduced its series of new Iphone's to the market, Nokia was quickly reduced to being a market follower. This was caused by Nokia's slow reaction to the market changes,

In our model firms strategically choose to hold cash for investment purposes, allowing them to preempt their competitors and thereby securing a higher market share in the future. This is supported by the empirical study of Fresard (2010) who shows how large, relative-to-rivals, cash reserves lead to systematic future market share gains at the expense of industry rivals. Thus cash policy comprises a substantial strategic element, which is also in line with the predatory behavior of firms studied by Bolton and Scharfstein (1990). In our model cash is particularly important for investments purposes for a firm facing a high level of preemption risk. Having cash increases the probability of securing sufficient financing for investment before competitors, and thereby the probability of firm growth.

The current literature on competition and corporate cash holdings has produced mixed empirical evidence. One part of the literature argues that more competition leads to lower or less certain cash flows increasing the risk of default due to illiquidity, and thereby also increasing the precautionary savings of firms (Morellec et al., 2014; Hoberg et al., 2014). However, other papers such as Grullon and Michaely (2007) argue that competitive pressure has a disciplinary effect on managers. Firms that are more exposed to competition are therefore likely to have higher payout ratios and lower corporate cash holdings, thereby reducing the agency costs of free cash flow. In this paper we focus on disentangling the effect of a certain type of competition, namely the effect of preemption risk on corporate cash holdings. We show that an increase in competition may lead to both an increase or a decrease in corporate cash holdings, which may help reconcile the mixed empirical findings. A related paper dealing with this issue is that of Lyandres and Palazzo (2014). In their paper the relation between competition and corporate cash holdings depend on the firm's level of financial constraints. They show that corporate cash holdings are decreasing with the intensity of competition for unconstrained firms and increasing for constrained firms. Whereas they focus on product-market competition in future output markets, we focus specifically on time-to-finance and preemption risk to show how cash holdings may be both increasing and decreasing with competition.

and in the end Nokia lost out not only to Apple, but also to HTC and Samsung who quickly followed in Apple's footsteps and launched new smart-phones themselves.

Previous studies have primarily relied on the cost of external capital to explain corporate cash behavior. In contrast we study how time-to-finance contributes to explaining the observed empirical variation. We setup a model in which the firm is not subject to direct costs when issuing capital, however, it is not able to obtain an unlimited amount of external financing at any point in time. Rather, the firm has to search for investors in the capital market when it requires funding. In praxis firms are not able to obtain an unlimited amount of financing at the spot. Raising capital through a seasoned equity offering (SEO) is often associated with a long period of implementation³. A SEO typically involves negotiating with investment banks followed by a due diligence process to certify the quality of the firm and the production of a prospectus. Finally the issue goes through a book building process in which the lead underwriter and the management of the firm needs to search for investors until there is sufficient demand for the issue to go through.

While time-to-finance is important for public firms, it may be even more prevalent for private firms. Private firms need to search for new investors such as venture capital firms or angel investors to enable growth, and this process is often long and has a significant risk of failure. Hence, search frictions in capital markets should be considered an important aspect of the financial policies of both public and private firms. By introducing time-to-finance we are able to show how a firm's liquidity management policy depends on the industry characteristics of competition. Since a firm facing preemption risk has less time to raise cash in the capital market, it hedges the risk of losing the growth option by retaining large buffers of cash within the firm. In our model the firm optimally balance the cost of holding cash with the risk of losing out on an investment opportunity before securing sufficient financing.

The paper is structured as follows. Section 2 relates the paper to the literature. Section 3 presents the model and the underlying assumptions. Section 4 describes the solution procedure. Section 5 presents the model implications and discuss the results, and Section 6 concludes. All proofs are gathered in the Appendix.

³Gao and Ritter (2010) show that firms choose to undertake a fully marketed offer with higher fees, to reduce the time of the book building process.

2 Ties to the literature

Our work is related to several strings of literature, firstly to the literature on financial constraints and their effects on corporate cash holdings (e.g Almeida et al., 2004; Acharya et al., 2007; Gryglewicz, 2011; Anderson and Carverhill, 2012; Kisser, 2013; Harford et al., 2013; Qiu and Wan, 2015). More specifically we relate to the part of the literature focusing on the effect of time frictions in capital markets. A closely related paper is that of Hugonnier et al. (2015) who show how capital supply uncertainty leads to a shift in the optimal payout policy of a firm from a traditional barrier strategy to a band strategy. In contrast to their paper, we show that firms are likely to hold cash for investment purposes, that time-to-finance does not have a uniform effect on corporate cash holdings for all firms, and that the effect is determined by the industry characteristics of competition. This result helps explain the empirical evidence on the relationship between R&D intensity and corporate cash holdings (e.g. Schroth and Szalay, 2007; Bates et al., 2009; Schroth and Szalay, 2010; Brown and Petersen, 2011; Brown et al., 2012). The underlying reason is that R&D intensity and preemption risk are likely to be highly correlated. Another related paper is that of Ma et al. (2014), who set up a general equilibrium model of R&D competition to show how cash plays a strategic role when there is a significant winner's advantage in innovations and outside financing takes time. In contrast to their paper, we set up a model focusing on the individual firms optimal liquidity management policy, and show how the role of corporate cash changes when firms are exposed to time-to-finance and preemption risk.

Secondly, our paper relates to the literature analyzing the effect of competition on corporate cash holdings (e.g Grullon and Michaely, 2007; Hoberg et al., 2014; Morellec et al., 2014). While these papers primarily focus on determining the relation between product-market competition and corporate cash holdings, we study another type of competition, namely the fight for patents and acquisitions. Moreover, our model provides an explanation for the mixed empirical evidence on the relation between corporate cash holdings and competition by showing that cash is hump-shaped in competition in the presence of time-to-finance. A closely related

paper is that of Lyandres and Palazzo (2014) who argue that the relation between corporate cash holdings and competition in future output markets depend on the level of financial constraints. They argue that cash holdings are decreasing with the intensity of competition for unconstrained firms and increasing for constrained firms. In contrast to their paper we focus specifically on time frictions in capital markets and preemption risk to show how cash holdings may be both increasing and decreasing with competition.

Lastly, our paper relates to the string of literature focusing on the effect of competition on the firm's investment decision. Lambrecht and Perraudin (2003) provides evidence on how the threat of preemption and incomplete information affects the investment decision of firms. They show that the optimal investment threshold may lie anywhere between the zero-NPV investment rule and the standard non-strategic investment threshold proposed by McDonald and Siegel (1986) and further developed by Dixit and Pindyck (1994). Mason and Weeds (2010) show that firms are willing to give up some option value to obtain the role as market leader in all subsequent periods. Hence, the threat of preemption forces firms to invest in-optimally compared to the standard non-strategic investment threshold. Our paper supports the above results by showing how the investment decision of the firm approaches the zero-NPV trigger when the risk of preemption increases.

3 The Model

We consider an all equity financed firm with assets-in-place generating stochastic earnings⁴. Denote the cumulative earnings at time t as Y_t and the instantaneous earnings rate as dY_t . We assume that the cumulative earnings evolve according to the following process

$$dY_t = \mu_0 dt + \sigma dZ_t \tag{1}$$

⁴The model can be extended to include both debt and equity issuance. However, this does not change the qualitative results of the paper.

where μ_0 is the expected earnings rate, σ represents the volatility and Z_t is a standard Brownian motion under the risk-neutral probability measure. It is assumed that the firm has an opportunity to invest in a project which raises the future expected earnings rate from μ_0 to μ_1 where $\mu_0 < \mu_1$. If the firm invests it must pay an initial lump-sum investment cost I . We assume that after investing the firm does not receive any further growth options⁵.

Further, it is assumed that the firm has the option to liquidate at any time. If the firm liquidates, shareholders are left with the liquidation value $l_i = \frac{\alpha\mu_i}{r}$, where $i \in \{0, 1\}$. Here l_0 and l_1 denotes the liquidation value before and after investment respectively. The liquidation value is given a fraction α of the risk free value of assets-in-place, where $0 \leq \alpha \leq 1$. The term $(1 - \alpha)\frac{\mu_i}{r}$ can therefore be interpreted as the liquidation cost. We denote the time of investment as T and the time of liquidation as τ_l .

The firm is subject to competition and faces the risk of being preempted by a competitor. In case of preemption the growth option disappears and the firm is instantaneously liquidated⁶. We follow Li and Mauer (2014) in applying the reduced form modeling of preemption risk by a Poisson process with a constant exogenous mean arrival rate λ , and denote the time at which the firm is preempted as τ_p . When $\lambda = 0$ the firm has a perpetual growth option and faces a classic real options problem. In contrast when $\lambda \rightarrow \infty$ the firm faces a now or never decision of investment upon receiving the growth option.

One way to cover the investment cost is by drawing on internal cash holdings. To accumulate internal cash the firm may choose to retain its earnings and to invest them in a liquid risk-free security earning the risk-free rate r . We assume that there is a cost associated with managing the liquidity reserves within the firm, i.e. a cost of carry, denoted δ . This could also be interpreted as an agency cost of free cash flow as in Jensen (1986), due to the managers pos-

⁵The model could be extended to multiple rounds of investment opportunities, however we refrain from presenting such a version here as it only complicates the analysis without changing the qualitative results.

⁶This assumption can be relaxed by allowing the firm to continue operations at a lower level of expected earnings $\mu_P \leq \mu_0$. While this assumption may seem strict, it is primarily for technical reasons, as it allows for closed form solutions of large parts of the model without changing the qualitative results.

sibility of extracting private benefits from empire building or other value destroying activities. The costs associated with holding cash implies that it is optimal for the firm to start paying dividends when cash balances become too large.

The firm may also choose to increase its cash holdings by issuing an amount of equity f and to sell it to investors in the capital market. In contrast to large parts of the existing liquidity management literature we assume that the firm is not able to obtain an unlimited amount of financing at any point in time. More specifically our modeling follows that of Hugonnier et al. (2015) by assuming that it takes time to secure outside financing due to capital supply uncertainty. The firm therefore has to search for investors in the capital market when in need of external funds, hence there is a time-to-finance (Ma et al., 2014). For simplicity we assume that there is no direct cost of external financing. Outside investors are capable of providing all the financing needed by the firm, and they arrive according to a Poisson process N with a constant exogenous arrival rate $\Phi \geq 0$. Hence, the probability of meeting an investor over the time interval $[t, t + dt]$ is given by Φdt and the expected time-to-finance is given by $1/\Phi$. All of the above assumptions yield the evolution of the firm's stock of cash as

$$\begin{aligned}
dc_t = & ((r - \delta)c_t + \mu_0)dt + \sigma dZ_t - dD_t + f_t dN_t \\
& - 1_{\{t=T\}}I + 1_{\{t, \tau_p \geq T\}}(\mu_1 - \mu_0)dt
\end{aligned} \tag{2}$$

where D_t represents the cumulative payouts to shareholders and f is a nonnegative process representing the financing raised by the firm. From this we see that the firm's cash holdings are increasing with the expected earnings, the interest earned on current cash holdings net of cost-of-carry and the outside financing, and decreasing in the payouts made to shareholders and the investment cost. It is assumed that the firm is liquidated if it is unable to cover its current obligations, i.e. if the cash holdings reach 0. In the remainder of the paper we use the term cash, cash holdings or cash buffer interchangeably, to represent all holdings of liquid risk-free securities and denote this c .

The life time of the firm can be separated into two stages, which is illustrated in Figure 1.

In the first stage, the firm is in possession of a growth option which can be financed either internally or externally. The firm is subject to time-to-finance, and therefore has to search for investors in the capital market if it chooses to rely on external financing. If the firm successfully invests it enters the second stage, in which it continues operations without further growth options until liquidation. However, the firm is subject to competition in the first stage and faces the risk of being preempted. If preemption occurs before the investment, the firm is liquidated and does not enter the second stage.

The firm chooses the optimal payout, investment, financing and cash management policies to maximize the value of incumbent shareholders. We denote the value of the firm as $V_i(c)$ for $i \in \{0, 1\}$ where $V_0(c)$ denotes the value of the firm before investment and $V_1(c)$ is the value after investment for different levels of cash holdings:

$$V_i(c) = \sup_{dD_t, f, T} \mathbb{E}_c \left[\int_0^{\tau_l} e^{-rt} (dD_t - f_t dN_t) + e^{-r\tau_l} (l_0 + 1_{\{\tau_l \wedge \tau_p > T\}} (l_1 - l_0)) \right] \quad (3)$$

Hence, the firm maximizes the present value of future dividends to incumbent shareholders by choosing the optimal dividend (dD_t), financing (f) and investment policy (T). The first term represents the present value of dividend payments made to incumbent shareholders until the liquidation time τ_l , net of new investors claim on the future earnings. The second term represents the value accruing to shareholders in liquidation. Liquidation may occur either either due to preemption prior to investment, or due to illiquidity which may happen before or after the investment.

As there are no costs associated with obtaining or searching for external financing, it is optimal for the firm to always search for investors. The firm issues as much equity as needed to cover the investment cost and to secure the optimal level of cash holdings post investment. For tractability reasons we assume that the firm does not contract on an exact issuance amount prior to searching. Rather, the firm is able to continuously adjust the issuance amount f in response to changes in its cash holdings. We further assume that investors in the capital market are facing a zero net present value investment decision, when deciding whether to inject capital

into the firm or not. Hence, all value gains from the equity issuance accrues to the existing shareholders⁷. We restrict our analysis to cases when the investment cost I is sufficiently low to ensure that the investment opportunity has a positive net present value (NPV), which makes both the issuance and investment decisions trivial. Hence, the firm always wants to invest when finding an investor in the capital market and issues sufficient equity to cover the investment cost and to reach the post investment optimal level of cash. This implies that the firm only needs to determine its optimal payout policy post and prior to investment, that is, to optimize over dD_t .

4 Solution Procedure

In this section we solve the firm's optimization problem by determining the optimal financial policy of the firm and the corresponding firm value. To do so, we solve the model backwards by first determining the optimal payout policy and firm value after investment has occurred. Thereafter, we use these results to determine the optimal payout policy and the value of a firm that has a growth option and is subject to time-to-finance and preemption risk.

4.1 The value of the firm after the investment

A firm with no growth options may choose to either pay dividends, to retain earnings and search for investors, or to be liquidated. Because the marginal cost of holding cash is constant and the marginal benefit is decreasing, we assume that there exist an optimal level of cash which we denote C_1^* . At this point the marginal cost and benefit of retaining cash are equal, and it is optimal to start paying dividends. The value of a firm with cash holdings that exceed the optimal cash target is given by:

$$V_1(c) = c - C_1^* + V_1(C_1^*), \text{ for } c \geq C_1^* \quad (4)$$

⁷This corresponds to a Nash bargaining game in which existing shareholders have all the bargaining power. One could relax this assumption and follow the modeling of Hugonnier et al. (2015). However, as this is not our main focus, and does not alter the qualitative results of our analysis we refrain from doing so.

Hence, if the firm has an initial level of cash above the target it should optimally distribute all cash holdings above C_1^* as dividends to shareholders. When cash holdings are below C_1^* it is optimal for the firm to retain all earnings and search for investors in the capital market to bring the cash holdings to a level of C_1^* .

The value function of a firm with no growth option satisfies the following ordinary differential equation (ODE) over the interval $c \in [0, C_1^*)$.

$$rV_1(c) = ((r - \delta)c + \mu_1)V_1'(c) + \frac{\sigma^2}{2}V_1''(c) + \Phi[V_1(C_1^*) + c - C_1^* - V_1(c)] \quad (5)$$

The left-hand side represents the risk-free rate of return. The right-hand side represents the expected change in firm value in the region where the firm does not pay dividends. We refer to this as the retention region. The first and second term represents the effect of retaining cash within the firm and the earnings volatility respectively. The third term represents the effect of time-to-finance on firm value. This is given as the product of the probability of finding an investor and the surplus accruing to incumbent shareholders when raising cash holdings from c to C_1^* .

To determine the value function, equation (5) has to be solved subject to the appropriate boundary conditions. The lower boundary is given as the value at liquidation l_1 . At the upper boundary, the firm faces a reflective barrier problem. Hence, we need to solve for the policy that satisfies both the value-matching, smooth-pasting and super contact condition⁸. This lead to

⁸As we are dealing with a barrier control problem the first-order or smooth-pasting condition holds for any position of the barrier, and the second-order derivative or super contact condition characterizes the optimal barrier (Dixit, 1993).

the following set of boundary conditions:

$$V_1(0) = l_1 \quad (6)$$

$$V_1(c) = V_1(C_1^*) + c - C_1^*, \text{ for } c \geq C_1^* \quad (7)$$

$$\lim_{c \uparrow C_1^*} V_1'(c) = 1 \quad (8)$$

$$\lim_{c \uparrow C_1^*} V_1''(c) = 0 \quad (9)$$

Hence, at the dividend boundary C_1^* the value function must merge smoothly with the value function of the dividend paying firm represented by (4). To characterize the solution of the problem we introduce the following notation. Let

$$F_1(c) = M(a, b, z(c)) \quad (10)$$

$$G_1(c) = \frac{(r - \delta)c + \mu_1}{\sqrt{(r - \delta)\sigma}} M(a + 1 - b, 2 - b, z(c)) \quad (11)$$

where $a \equiv -\frac{\Phi+r}{2(r-\delta)}$, $b \equiv \frac{1}{2}$, $z(c) \equiv -\frac{((r-\delta)c+\mu_1)^2}{(r-\delta)\sigma^2}$ and M is the confluent hyper-geometric function. The solution is characterized in the following proposition:

Proposition 1 *The optimal target level of cash C_1^* which maximizes the value of the firm with no future growth options V_1 is the solution to*

$$\gamma(C_1^*)F_1(0) + \omega(C_1^*)G_1(0) + \frac{\Phi}{\Phi + r} \left(\frac{\delta C_1^* + \mu_1}{r} + \frac{\mu_1 + (r - \delta)c}{\Phi + \delta} \right) = l_1 \quad (12)$$

where the functions $\gamma(c)$, $\omega(c)$ are defined by

$$\gamma(c) = \frac{-G'''(c)\delta e^{((r-\delta)c+\mu_1)^2/((r-\delta)\sigma^2)}}{2\sigma^{-3}\sqrt{(r-\delta)}(\Phi+\delta)(\Phi+r)} \quad (13)$$

$$\omega(c) = \frac{F''(c)\delta e^{((r-\delta)c+\mu_1)^2/((r-\delta)\sigma^2)}}{2\sigma^{-3}\sqrt{(r-\delta)}(\Phi+\delta)(\Phi+r)} \quad (14)$$

For any level of cash $c < C_1^*$, the value function of a firm with no growth option is

$$V_1(c) = \gamma(C_1^*)F_1(c) + \omega_1(C_1^*)G_1(c) + \frac{\Phi}{\Phi + r} \left(V_1(C_1^*) + c - C_1^* + \frac{\mu_1 + (r - \delta)c}{\Phi + \delta} \right) \quad (15)$$

where the firm value at the optimal cash target C_1^* satisfies

$$V_1(C_1^*) = \frac{(r - \delta)C_1^* + \mu_1}{r}$$

Figure 2 plots the value of the firm after the investment is made and the corresponding marginal value of cash for the base case parameterization. The set of parameters chosen are similar to that of Morellec et al. (2014) and Hugonnier et al. (2015), and is listed in Table 1. The vertical line represents the optimal target level of cash $C_1^* = 0.31$. The black solid line on the left hand side of C_1^* represents the value of the existing shareholders given by the solution to equation (5) subject to the boundary conditions. The firm value is equal to l_1 when cash holdings reach zero and the firm chooses to initiate payout if cash holdings go above C_1^* . The dashed line on the right hand side of C_1^* represents the value of the firm if the initial cash holdings are above the optimal cash target. The red line represents the marginal value of cash which is decreasing in c .

Since the firm has no future growth options, cash is held only to hedge against illiquidity risk. As cash holdings increase, the probability of default due to illiquidity decreases and cash holdings become less valuable. As there are no taxes in the model, the firm always has an outside option to distribute dividends to shareholder with a marginal of value equal to one. When the marginal value of retaining cash within the firm becomes less than one the firm therefore chooses to payout excess cash as dividends. This results in the marginal value of cash being equal to one for all levels of cash above C_1^* , which is represented by the red dashed line. The results shown in Figure 2 are in line with the existing literature on liquidity management (e.g. Anderson and Carverhill, 2012). The firm sets an optimal target level of cash and chooses to retain all earnings when cash holdings are below the target, and pays out all earnings when cash holdings reach the target.

4.2 The value of the firm with a growth option in place

We now turn our focus to the solution of the optimal payout policy when the firm has a growth option in place. To determine the optimal policy we need to analyze the following two scenarios: 1) The firm funds the investment cost using both internal cash and external financing or, 2) the investment is funded using external financing only. We start by assuming that the firm relies on both internal and external financing. Hence, for $c \in [0; C_U^*)$ the firm retains earnings and searches for investors in the capital market, where C_U^* denotes the optimal investment trigger and $U(c)$ is the corresponding value function. The value of the firm can be found as the solution to the following ODE subject to the appropriate boundary conditions.

$$\begin{aligned} rU(c) = & ((r - \delta)C_t + \mu_0) U'(c) + \frac{\sigma^2}{2} U''(c) \\ & + \Phi [V_1(C_1^*) + c - C_1^* - I - U(c)] + \lambda [V_P(c) - U(c)] \end{aligned} \quad (16)$$

The first and second term on the right-hand side represent the expected change in value due to changes in savings and earnings volatility respectively. The third term represents the change in value due to the arrival of outside investors. When the firm finds investors in the capital market it issues sufficient capital to invest and to hold the optimal post-investment level of cash holdings C_1^* . Hence, the incumbent shareholders receive an increase in value equal to the value gain of investing minus the claim to new investor. This can be written as $V_1(C_1^*) + c - C_1^* - I - U(c)$. The last term represents the effect of preemption risk on the value of the existing shareholders, given as the product between the probability of default and the value loss caused by preemption.

If the firm relies both on internal and external financing there exists an optimal level of cash C_U^* at which the firm chooses to invest using internal cash holdings only. The value of the firm with cash holdings above C_U^* is equal to the value after investing at a level of cash equal to $c - I$. We can express this as

$$U(c) = V_1(c - I), \text{ for } c \geq C_U^* \quad (17)$$

The investment policy C_U^* and the corresponding firm value are given by the solution to equation (16) subject to the following boundary conditions.

$$U(0) = l_0 \quad (18)$$

$$U(c) = V_1(c - I), \text{ for } c \geq C_U^* \quad (19)$$

$$U'(C_U^*) = V_1'(C_U^* - I) \quad (20)$$

This leads to the following proposition:

Proposition 2 *The value of the firm relying on both internal and external financing can be written as*

$$U(c) = \begin{cases} \gamma_u(C_U^*)F_0(c) - \omega_u(C_U^*)G_0(c), & c \leq C_U^* \\ V_1(c - I), & c > C_U^* \end{cases} \quad (21)$$

The constant C_U^* is the unique solution to

$$\gamma_u(C_U^*)F_0(0) + \omega_u(C_U^*)G_0(0) = l_0 - \Theta(0),$$

where

$$\Theta(c) = \frac{\Phi + \lambda}{\Phi + \lambda + r} \left(\frac{(r - \delta)c + \mu_0}{\Phi + \lambda + \delta} + c \right) + \frac{\Phi(V_1(C_U^*) - C_U^* - I)}{\Phi + \lambda + r} + \frac{\lambda V_p(c)}{\Phi + \lambda + r} \quad (22)$$

and the constants γ_u and ω_u are defined in Appendix A.2.

Using the base case parameter values, Figure 3 illustrates the solution by plotting the value of the firm that is relying on both internal and external financing. It can be seen that the optimal investment threshold C_U^* lies above the actual investment cost. Since the firm faces the risk of illiquidity after investing due to time-to-finance, it invests later than a firm not subject to capital market frictions. Hence, the firm builds a sufficiently large cash buffer to cover the investment cost and reduce the future risk of default.

From Figure 4 we see that the marginal value goes below one for some levels of $c \leq C_U^*$. As the firm always has the option to pay out the marginal dollar as dividends, there may exist a region in which it is optimal for the firm to make an intermediate dividend payment to shareholders. To determine if such a strategy exists we need to analyze whether it is optimal to switch strategies for any level of cash holdings. Denote C_H^* as the optimal point of investment and C_L^* the point at which the firm switches from relying on both internal and external financing to relying on external financing only. Since the optimal switching trigger must depend on the firm value of both strategies we need to determine the optimal payout policy and corresponding value of a firm relying only on external financing.

If the firm relies only on external financing for investment purposes, internal cash is reduced to a hedging tool against illiquidity risk. The solution to this problem is therefore similar to that of the firm after investment. Since the marginal benefit of cash is decreasing and the marginal cost of holding cash is constant there exists an optimal level of cash C_W^* at which the marginal cost and benefit of holding cash is equalized. Above this point it is optimal for the firm to pay out all earnings to shareholders in the form of dividends. Hence, the value of the firm for all level of cash above the optimal cash target C_W^* can be written as

$$W(c) = c - C_W^* + W(C_W^*), \text{ for } c \geq C_W^* \quad (23)$$

For all $c \in [0; C_W^*)$ the value of the firm and the optimal payout policy is given by the solution to equation (24), which is the same as equation (16),

$$\begin{aligned} rW(c) = & ((r - \delta)C_t + \mu_0) W'(c) + \frac{\sigma^2}{2} W''(c) \\ & + \Phi [V_1(C_1^*) + c - C_1^* - I - W(c)] + \lambda [V_P(c) - W(c)] \end{aligned} \quad (24)$$

subject to the following boundary and optimality conditions

$$W(0) = l_0 \quad (25)$$

$$W(c) = W(C_W^*) + c - C_W^* \text{ for } c \geq C_W^* \quad (26)$$

$$\lim_{c \uparrow C_W^*} W'(c) = 1 \quad (27)$$

$$\lim_{c \uparrow C_W^*} W''(c) = 0 \quad (28)$$

This leads to the following proposition:

Proposition 3 *The value of a firm relying only on external financing can be written as*

$$W(c) = \begin{cases} \gamma_w(C_W^*)F_0(c) + \omega_w(C_W^*)G_0(c), & c \leq C_W^* \\ c - C_W^* + W(C_W^*), & c > C_W^* \end{cases} \quad (29)$$

The constant C_W^* and C_U^* are the unique solutions to

$$\gamma_w(C_W^*)F_0(0) + \omega_w(C_W^*)G_0(0) = \gamma_u(C_U^*)F_0(0) + \omega_u(C_U^*)G_0(0) = l_0 - \Theta(0)$$

where

$$\Theta(c) = \frac{\Phi + \lambda}{\Phi + \lambda + r} \left(\frac{(r - \delta)c + \mu_0}{\Phi + \lambda + \delta} + c \right) + \frac{\Phi(V_1(C_1^*) - C_1^* - I)}{\Phi + \lambda + r} + \frac{\lambda V_p(c)}{\Phi + \lambda + r} \quad (30)$$

and the constants γ_w and ω_w are defined in Appendix A.3.

To complete the solution, we need to determine which strategy the firm should optimally follow for a given level of cash. Since cash holdings are associated with a constant cost-of-carry, and the marginal benefit of cash is U-shaped the decision of whether retain or not depends on the current level of cash. If the firm's cash holdings are far from the optimal investment level, it should be less inclined to rely on both internal and external financing, since the cost of retaining cash outweighs the benefits of an increased probability of investment. The firm should therefore optimally choose to wait for external financing as it is too costly to accumulate suf-

efficient internal cash. However, if the firm's cash holdings are close to the optimal investment threshold, it may be optimal to incur the cost of holding cash to ensure a high probability of investment. Figure 4 illustrates this point. The marginal value of cash is decreasing for low levels of cash and increasing for cash holdings close to the investment threshold. Denote the value function of the firm with a growth option in place $V_0(c)$. The firm chooses only to rely on outside financing to cover the investment costs when $c \leq C_L^*$ and is valued according to $V_0(c) = W(c)$. For $c \geq C_H^*$ the firm chooses to invest and the value equals the value after investing so that $V_0(c) = V_1(c - I)$. When $C_L^* \leq c \leq C_H^*$ the value function satisfies the following ODE:

$$\begin{aligned}
rV_0(c) = & ((r - \delta)C_t + \mu_0) V_0'(c) + \frac{\sigma^2}{2} V_0''(c) \\
& + \Phi [V_1(C_1^*) + c - C_1^* - I - V_0(c)] + \lambda [V_P(c) - V_0(c)]
\end{aligned} \tag{31}$$

Similar to above, the constants C_L^* and C_H^* can be determined from the following boundary conditions.

$$V_0(C_L^*) = W(C_L^*) \tag{32}$$

$$V_0'(C_L^*) = W'(C_L^*) \tag{33}$$

$$V_0(C_H^*) = V_1(C_H^* - I) \tag{34}$$

$$V_0'(C_H^*) = V_1'(C_H^* - I) \tag{35}$$

Condition (32) and (34) represents the value matching conditions at the upper and lower boundary, ensuring that the value functions merge at the points C_L^* and C_H^* . Conditions (33) and (35) requires that the value functions merge smoothly, and allow us to determine the value maximizing thresholds. The solution to the problem is characterized by the following proposition.

Proposition 4 *The value of the firm with a growth option in place can be written as:*

$$V_0(c) = \begin{cases} W(c), & \text{for } c \leq C_L^* \\ \gamma_u F_0(c) + \omega_u G_0(c) + \Theta(c), & \text{for } C_H^* \geq c \geq C_L^* \\ V_1(c - I), & \text{for } c \geq C_H^* \end{cases} \quad (36)$$

The optimal switching points $C_L^ \geq C_W^*$, $C_H^* \in [C_U^*, C_1^* + I]$, and the constants γ_u and ω_u are the unique solutions to conditions (32)-(35), and are given in Appendix A.4.*

Figure 5 illustrates the solution for the base parameterization in the case of no preemption risk (a), and the case of an expected time to preemption of 2 years, $\lambda = 0.5$ (b). The management choose to retain cash and search for investors when $c \leq C_W^*$. If c lies between C_W^* and C_L^* it is optimal to pay out a lump-sum dividend bringing the firm down to C_W^* . If cash holdings go above C_L^* the firm retains its earnings and searches for investors. If cash holdings reach C_H^* the firm invests and covers the cost with internal cash. The retention region below C_W^* can be interpreted as the cash held for hedging illiquidity risk and the region between C_L^* and C_H^* can be interpreted as the cash held for investment purposes.

From panel (a) in Figure 5 we see that the upper region is quite small. This supports the string of literature arguing that firms primarily use cash holdings to hedge illiquidity risk (e.g. Lins et al., 2010; Gryglewicz, 2011; Anderson and Carverhill, 2012; Hugonnier et al., 2015). However, the prior results are based on the assumption that investment opportunities are perpetual, i.e. the firm has an infinite time to wait for external financing. From panel (b) in Figure 5 we see that competition shifts the optimal boundaries. Most noticeably is the large increase in the upper retention region between C_L^* and C_H^* . Hence, competition increases the firm's incentive to retain cash for investment purposes. The result can be explained by returning to the marginal value of cash. As can be seen from Figure 4, the marginal value of cash is U-shaped, making the value function convex over a range of cash holdings. Preemption risk increases the marginal value of cash in the upper region and thereby also the range over which the value function is

convex, thereby increasing the retention region between C_L^* and C_H^* , which can be seen from panel (b). Hence, firms exposed to competition and time-to-finance are likely to value cash for investment and therefore hold more cash on their balance sheet.

5 Model Implications

In this section we turn to the implications of the model. We start by analyzing how time-to-finance affects the optimal payout, investment and switching boundaries. Secondly, we show how competition affects the financial policies and shifts the role of corporate cash holdings. Finally, we provide comparative statics to show how our results hold for a wide set of parameters. Table 2 reports the optimal benchmark payout, investment and switching thresholds with and without preemption risk.

Figure 6 depicts the relation between the optimal payout, investment and switching thresholds and the arrival rate of investors. When cash holdings are below C_W^* (the blue line) firms retain their earnings and search for investors. Between C_W^* and C_L^* it is optimal for the firm to payout cash as a lump-sum dividend, bringing them back to C_W^* . When cash holdings are between C_L^* and C_H^* the firm retains its earnings and searches for investors to fund the project. If the firm's cash holdings reach a level of C_H^* prior to meeting an investor, the firm invests and relies solely on internal funds to cover the investment cost. As an expansion of the retention regions, in particular the upper retention region, increases the possibility of higher cash holdings, an increase (decrease) in the total retention region can be interpreted as an increase (decrease) in corporate cash holdings.

In Figure 6 it is seen that both cash which is held with the purpose of hedging illiquidity as well as the cash held for investment purposes are decreasing with the arrival of investors Φ . This implies that the total retention region is increasing with time-to-finance $1/\Phi$. When time-to-finance decreases the firm has less need for cash to avoid default and to ensure investment prior to preemption. This leads to the following empirical prediction:

***Prediction 1:** Firms that are more exposed to time frictions in capital markets are likely to hold more cash.*

This prediction is in line with the traditional view on financial constraints. Firms that have a lot of experience in raising financing in the capital market are likely to have a more streamlined process and therefore also an easier access to capital. In our model this corresponds to a lower time-to-finance, leading the firm to hold less cash on its balance sheet.

Panel (a) in Figure 7 depicts the optimal investment, payout and switching thresholds as a function of preemption risk. This provides us with several interesting results. Firstly, we see that the optimal investment threshold is decreasing with preemption risk, which is in the line with the standard result that competition leads to over-investment (Lambrecht and Perraudin, 2003; Mason and Weeds, 2010). Secondly, we see that the area between C_L^* and C_H^* increases when the firm faces preemption risk, while the threshold C_W^* decreases. This is a key result of this paper. Higher levels of competition shift the role of corporate cash holdings away from hedging illiquidity risk, and make cash more valuable for investment purposes. This is in contrast to the results of Hugonnier et al. (2015) who argue that cash is primarily held for hedging illiquidity. The result of their paper can be seen from Figure 7, and corresponds to the case in which $\lambda = 0$. From the figure it is clear that even very low levels of competition dramatically change the outcome of the model. Setting $\lambda = 0.5$, which corresponds to an expected time to preemption equal to two years, increases the distance between C_L^* and C_H^* from 0.0427 to 0.3073. The fact that we do not need arbitrarily high levels of preemption risk to significantly affect the liquidity management policy indicate that our results are very robust and not a result of a limited set of parameters. Hence, the results suggest that competition makes cash holdings valuable for investments in line with the empirical evidence documented in e.g. Acharya et al. (2007) and Kisser (2013).

From panel (b) in Figure 7 we see that introducing preemption risk in general increases the total size of the retention regions. This leads to the following empirical prediction:

Prediction 2: *Firms exposed to preemption risk are more likely to have higher corporate cash holdings.*

Secondly, panel (b) provides us with additional insights on the relation between preemption risk and the size of the retention regions. For low and moderate levels of preemption risk, firms should be more inclined to hold cash to for investment purposes as competition increases. This is due to the fact that the benefit from the increased probability of investing before the competitor outweighs the cost of retaining cash. Hence, there is a positive relation between competition and cash held for investment. However, as preemption risk becomes large the relation reverts. The underlying reason for this is that it becomes too expensive to retain cash within the firm due to the high probability of losing the investment. Thus, the size of the upper retention region between C_L^* and C_H^* and the total retention region is hump-shaped in competition. Corporate cash holding should therefore be increasing for low levels of competition and decreasing for high levels of competition. This leads to the following empirical prediction:

Prediction 3: *Corporate cash holdings are hump-shaped in competition*

This result might help explain the mixed evidence on competition and corporate cash holdings found in the empirical literature (Grullon and Michaely, 2007; Hoberg et al., 2014; Morellec et al., 2014). Additionally, it can be seen from panel (b) that preemption risk decreases the cash held for hedging illiquidity. This is due to the fact that preemption risk decreases the value of the growth option and thereby the firm. As the firm decreases in value, shareholders are less willing to hold cash for hedging illiquidity risk. This result matches the existing results in the literature (see e.g. Gryglewicz, 2011).

Figure 8 illustrates the optimal investment, payout and switching thresholds as a function of earnings volatility, cost-of-carry, liquidation value and profitability of the investment opportunity. From panel (a) we see that the both the upper and lower retention region is increasing with earnings volatility. As earnings become more volatile the risk of illiquidity increases which leads the firm to increase the amount of cash held for hedging illiquidity represented by C_W^* (the blue line). Secondly, it can be seen that the optimal investment threshold C_H^* (the red

line) is increasing with earnings volatility. Since the risk of becoming illiquid after investment increases with earnings volatility, it is optimal for the firm to invest later using internal funds, as this ensures a sufficient buffer of liquidity after investment. The optimal investment threshold therefore approaches $I + C_1^*$ when σ increases. From panel (b) it can be seen that cash holdings are decreasing with the cost-of-carry. The intuition of this result is straight forward. As the cost of retaining cash increases and the benefits of cash remain constant, the firm will choose to decrease its cash holdings for both investment and hedging purposes. Hence, firms suffering from higher levels of agency problems should hold less cash.

In panel (c) we see that an increase in the liquidation value leads to a decrease in all of the thresholds. Firstly, as the firm become more valuable at liquidation the incentive to hedge illiquidity risk decreases, and the payout threshold C_W^* (the blue line) decreases. Secondly, it can be seen that the optimal investment threshold decreases. Since the level of expected earnings is higher after investment, an increase in the liquidation value has a larger positive effect on the value after investing, which leads the firm to invest sooner. Hence, more tangible firms should have lower cash balances. Panel (d) shows that an a higher increase in expected earnings from investment leads the firm to invest sooner. Secondly it is seen that, as the growth option becomes more valuable the firm chooses to hold more cash both to avoid default, and to ensure a higher probability of investing before its competitors. Hence, better investment opportunities leads to higher corporate cash holdings.

6 Conclusion

When external financing can not be obtained on the spot firms should optimally hold cash to avoid future default and underinvestment problems. The threat of underinvestment as a result of time-to-finance is especially large for firms with short lived investment opportunities. Hence, firms operating in industries with high levels of preemption risk should hold more cash to reduce underinvestment. In this paper we develop a dynamic model of investment and liquidity management in which firms face competition and time frictions in capital markets. We show

that corporate cash holdings are increasing in time-to-finance and that time-to-finance do not have a uniform effect on firms' financial policies. Rather the effect depends on the industry specific characteristics of competition. Specifically we show that time-to-finance has a significant effect on the liquidity management policy of firms that are exposed to preemption risk. In contrast to much of the existing literature, we show that cash is not only valuable for hedging illiquidity risk, but also for investment. Furthermore, we show that the results are robust and not caused by extreme parameter values. Even low levels of preemption risk significantly alter the firm's optimal liquidity management policy. Lastly, we show that corporate cash holdings are hump-shaped in competition, which may help reconcile the mixed evidence on the relation between corporate cash holdings and competition found in the literature.

Appendix

A.1 - Proof of Proposition 1

The solution to the ordinary differential equation

$$rV_1(c) = ((r - \delta)c + \mu)V_1'(c) + \frac{\sigma^2}{2}V_1''(c) + \Phi [V_1(C_1^*) - C^* + c - V_1(c)], \quad (37)$$

can be found by first finding a solution to the homogeneous part of the equation which can be written as

$$(\Phi + r)V_1(c) = ((r - \delta)c + \mu)V_1'(c) + \frac{\sigma^2}{2}V_1''(c). \quad (38)$$

We now show how the homogeneous part of the ODE can be rewritten to Kummer's equation for $g(\cdot)$.

$$z(c)g''(z(c)) + (b - z(c))g'(z(c)) - ag(z(c)) = 0$$

Define the transformation equation $V_1(c) = g(z(c)) = g\left(-\frac{[(r-\delta)c + \mu_1]^2}{(r-\delta)\sigma^2}\right)$, and

$$z(c) \equiv -\frac{[(r-\delta)c + \mu_1]^2}{(r-\delta)\sigma^2}.$$

Using this we can express the first and second order differentials as

$$\begin{aligned} V_1'(c) &= g'(z(c)) = -\frac{2((r-\delta)c + \mu_1)}{(r-\delta)\sigma^2}(r-\delta)g'(z(c)) = \frac{-2((r-\delta)c + \mu_1)}{\sigma^2}g'(z(c)) \\ V_1''(c) &= g''(z(c)) = \frac{2(r-\delta)}{\sigma^2}g'(z(c)) + \left(-\frac{2((r-\delta)c + \mu_1)}{\sigma^2}\right)^2 + g''(z(c)) \\ &= \frac{2(r-\delta)}{\sigma^2}g'(z(c)) + \frac{4(r-\delta)}{\sigma^2}z(c)g''(z(c)) \end{aligned}$$

Inserting this into the homogeneous part of the ODE given by (38) we get

$$\begin{aligned} (\Phi + r)g(z(c)) &= ((r-\delta)c + \mu_1) \left(\frac{-2((r-\delta)c + \mu_1)}{\sigma^2}g'(z(c)) \right) \\ &\quad + \frac{\sigma^2}{2} \left(\frac{2(r-\delta)}{\sigma^2}g'(z(c)) + \frac{4(r-\delta)}{\sigma^2}z(c)g''(z(c)) \right) \end{aligned}$$

which can be rearranged as

$$\frac{\Phi + r}{2(r-\delta)}g(z(c)) = -z(c)g'(z(c)) + \frac{1}{2}g'(z(c)) + z(c)g''(z(c))$$

Rewriting this further gives us the following

$$z(c)g''(z) + (b - z(c))g'(z(c)) - ag(z(c)) = 0$$

Which is Kummer's equation with a and b defined as

$$\begin{aligned} a &\equiv -\frac{\Phi + r}{2(r-\delta)} \\ b &\equiv \frac{1}{2} \end{aligned}$$

A numerical satisfactory solution of the Kummer equation near the origin is given by the combination of two linear independent functions

$$V_1(c) = \gamma F_1(c) + \omega G_1(c)$$

where the functions $F_1(c)$ and $G_1(c)$ are given by

$$\begin{aligned} F_1(c) &= M(a, b, z(c)) \\ G_1(c) &= \frac{(r - \delta)c + \mu_1}{\sqrt{(r - \delta)\sigma}} M(a + 1 - b, 2 - b, z(c)) \end{aligned}$$

and $M(\cdot)$ is the Kummer function of the first kind (Abramowitz and Stegun, 1964). We can now complete the solution by finding the particular solution for inhomogeneous part of the ODE. As the inhomogeneous part is linear in c we conjecture the following solution

$$\mathbb{V}(c) = \mathbb{A}c + \mathbb{B}. \quad (39)$$

Inserting this, along with the first and second order derivatives, into equation (6) we get

$$(\Phi + r)(\mathbb{A}c + \mathbb{B}) = ((r - \delta)c + \mu_1)\mathbb{A} + \Phi [V_1(C_1^*) + c - C_1^*] \quad (40)$$

For this to hold for all c we need \mathbb{A} and \mathbb{B} to satisfy the following relations

$$\begin{aligned} \mathbb{A}((\Phi + r) - (r - \delta)) - \Phi &= 0 \\ \mathbb{B}(\Phi + r) - \mu_1\mathbb{A} - \Phi(V_1(C_1^*) - C_1^*) &= 0 \end{aligned}$$

which gives us

$$\begin{aligned} \mathbb{A} &= \frac{\Phi}{\Phi + \delta} \\ \mathbb{B} &= \frac{\Phi}{\Phi + r} \left(\frac{\mu_1}{\Phi + \delta} + V_1(C_1^*) - C_1^* \right) \end{aligned}$$

Hence, the approximate solution to equation can be written as,

$$V_1(c) = \gamma F_1(c) + \omega G_1(c) + \frac{\Phi}{\Phi + r} \left(\frac{\mu_1 + (r - \delta)c}{\Phi + \delta} + V_1(C_1^*) + c - C_1^* \right)$$

where the two first terms are the solution to the homogeneous part and third term is the solution to the inhomogeneous part of the ordinary differential equation. To determine γ and ω we need to apply the specific boundary conditions. By applying the smooth-pasting and high-contact condition equivalent to condition (8) and (9), we get the following

$$\begin{aligned} \gamma F_1'(c) + \omega G_1'(c) + \frac{\Phi}{\Phi + \delta} &= 1, \\ \gamma F_1''(c) + \omega G_1''(c) &= 0. \end{aligned}$$

Solving the two equations gives us the functions γ and ω as:

$$\gamma(c) = \frac{\delta}{\Phi + \delta} \frac{-G_1''(c)}{F_1''(c)G_1'(c) - F_1'(c)G_1''(c)}, \quad (41)$$

$$\omega(c) = \frac{\delta}{\Phi + \delta} \frac{F_1''(c)}{F_1''(c)G_1'(c) - F_1'(c)G_1''(c)}. \quad (42)$$

We can use the following relations between the two linearly independent solutions $F_1(c)$ and $G_1(c)$, which follows from Abel's Identity and the fact that $F_1(c)$ and $G_1(c)$ solve equation (6) (see Hartman, 1964; Hugonnier et al., 2015)

$$F_1'(c)G_1(c) - F_1(c)G_1'(c) = -\frac{\sqrt{(r - \delta)}}{\sigma} e^{-((r - \delta)c + \mu_1)^2 / ((r - \delta)\sigma^2)} \quad (43)$$

$$F_1''(c)G_1(c) - F_1(c)G_1''(c) = \frac{2\sqrt{(r - \delta)}}{\sigma^3} ((r - \delta)c + \mu_1) e^{-((r - \delta)c + \mu_1)^2 / ((r - \delta)\sigma^2)} \quad (44)$$

$$F_1''(c)G_1'(c) - F_1'(c)G_1''(c) = \frac{2\sqrt{(r - \delta)}}{\sigma^3} r e^{-((r - \delta)c + \mu_1)^2 / ((r - \delta)\sigma^2)} \quad (45)$$

Hence, we can rewrite γ and ω as:

$$\gamma(c) = \frac{-G''(c)\delta e^{((r-\delta)c+\mu_1)^2/((r-\delta)\sigma^2)}}{2\sigma^{-3}\sqrt{(r-\delta)}(\Phi+\delta)(\Phi+r)}, \quad (46)$$

$$\omega(c) = \frac{F''(c)\delta e^{((r-\delta)c+\mu_1)^2/((r-\delta)\sigma^2)}}{2\sigma^{-3}\sqrt{(r-\delta)}(\Phi+\delta)(\Phi+r)}. \quad (47)$$

A.2 - Proof of Proposition 2

The solution to the ordinary differential equation (48) can be found by using similar arguments as in the proof of Proposition 1.

$$\begin{aligned} rU(c) = & ((r-\delta)c + \mu_0)U'(c) + \frac{\sigma^2}{2}U''(c) + \lambda[V_p(c) - U(c)] \\ & + \Phi[V(C_1^*) + c - I - C_1^* - U(c)] \end{aligned} \quad (48)$$

The homogeneous part of the equation can be written as

$$(\Phi + \lambda + r)U(c) = ((r-\delta)c + \mu)U'(c) + \frac{\sigma^2}{2}U''(c). \quad (49)$$

To find a solution we start by noting that this can be rewritten on the form of Kummer's equation for $g(\cdot)$. Define the transformation equation $U(c) = g(z(c)) = g\left(-\frac{[(r-\delta)c+\mu_0]^2}{(r-\delta)\sigma^2}\right)$, and

$$z(c) \equiv -\frac{[(r-\delta)c + \mu_0]^2}{(r-\delta)\sigma^2}.$$

We get the following equation

$$z(c)g''(z(c)) + (b - z(c))g'(z(c)) - ag(z(c)) = 0$$

Which is Kummer's equation with a and b defined as

$$\begin{aligned} a & \equiv -\frac{\Phi + \lambda + r}{2(r-\delta)} \\ b & \equiv \frac{1}{2} \end{aligned}$$

To find a particular solution for the inhomogeneous part of the ODE we conjecture the following solution:

$$\Theta(c) = \mathbb{A}c + \mathbb{B} \quad (50)$$

inserting this and its first and second order derivatives into equation (48) we get

$$(r + \lambda + \Phi)(\mathbb{A}c + \mathbb{B}) = ((r - \delta)c + \mu_0)\mathbb{A} + \Phi [V_1(C_1^*) + c - C_1^* - I] + \lambda V_p(c) \quad (51)$$

For this to hold for all c we need \mathbb{A} and \mathbb{B} to satisfy the following relations

$$\begin{aligned} \mathbb{A}((\Phi + \lambda + r) - (r - \delta)) - \Phi - \lambda &= 0 \\ \mathbb{B}(\Phi + \lambda + r) - \mu_0\mathbb{A} - \Phi(V_1(C_1^*) - C_1^* - I) - \lambda V_p(c) &= 0 \end{aligned}$$

which gives us

$$\begin{aligned} \mathbb{A} &= \frac{\Phi + \lambda}{\Phi + \lambda + \delta} \\ \mathbb{B} &= \frac{\Phi + \lambda}{\Phi + \lambda + r} \left(\frac{\mu_0}{\Phi + \lambda + \delta} \right) + \frac{\lambda V_p(c)}{\Phi + \lambda + r} + \frac{\Phi}{\Phi + \lambda + r} (V_1(C_1^*) - C_1^* - I) \end{aligned}$$

Hence, the general solution of equation (48) can be approximated by

$$U(c) = \gamma_u F_0(c) + \omega_u G_0(c) + \Theta(c)$$

where the third term is the solution to the inhomogeneous part which can be written as.

$$\begin{aligned} \Theta(c) &= \frac{\Phi + \lambda}{\Phi + \lambda + r} \left(\frac{\mu_0 + (\Phi + \lambda + r)c}{\Phi + \lambda + \delta} \right) + \frac{\lambda V_p(c)}{\Phi + \lambda + r} \\ &\quad + \frac{\Phi}{\Phi + \lambda + r} (V_1(C_1^*) - C_1^* - I) \end{aligned}$$

and γ_u and ω_u are the constants to be determined. By applying condition (19) and (20) we get the following two equations

$$\begin{aligned}\gamma_u F_0(c) + \omega_u G_0(c) + \Theta(c) &= V_1(c - I), \\ \gamma_u F'_0(c) + \omega_u G'_0(c) + \Theta'(c) &= V'_1(c - I).\end{aligned}$$

Which gives us the following expressions for γ_u and ω_u

$$\gamma_u(c) = \frac{(V_1(c - I) - \Theta(c))G(c) - (V'_1(c - I) - \Theta'(c))G'_0(c)}{F'_0(c)G_0(c) - F_0(c)G'_0(c)}, \quad (52)$$

$$\omega_u(c) = \frac{(V_1(c - I) - \Theta(c))F(c) - (V'_1(c - I) - \Theta'(c))F'_0(c)}{F'_0(c)G_0(c) - F_0(c)G'_0(c)}. \quad (53)$$

A.3 - Proof of Proposition 3

The solution to the ordinary differential equation (54) follows from the proofs of Proposition 1 and 2.

$$\begin{aligned}rW(c) &= ((r - \delta)c + \mu_0)W'(c) + \frac{\sigma^2}{2}W''(c) + \lambda[V_p(c) - W(c)] \\ &+ \Phi[V(C_1^*) + c - I - C_1^* - W(c)]\end{aligned} \quad (54)$$

The homogenous part of the equation can be rewritten to the Kummer equation

$$z(c)g''(z) + (b - z(c))g'(z(c)) - ag(z(c)) = 0$$

with a , b and $z(c)$ defined as

$$\begin{aligned}a &\equiv -\frac{\Phi + \lambda + r}{2(r - \delta)}, \\ b &\equiv \frac{1}{2}, \\ z(c) &\equiv -\frac{[(r - \delta)c + \mu_0]^2}{(r - \delta)\sigma^2}.\end{aligned}$$

The inhomogeneous part of equation (54) is equal to that of equation (48) and is therefore given as

$$\Theta(c) = \frac{\Phi + \lambda}{\Phi + \lambda + r} \left(\frac{\mu_0 + (\Phi + \lambda + r)c}{\Phi + \lambda + \delta} \right) + \frac{\lambda V_p(c)}{\Phi + \lambda + r} + \frac{\Phi}{\Phi + \lambda + r} (V_1(C^*) - C_1^* - I)$$

We can therefor write the general solution as

$$W(c) = \gamma_w F_0(c) + \omega_w G_0(c) + \Theta(c)$$

where γ_w and ω_w are constants to be determined. By applying condition (27) and (28) we get the following expressions

$$\gamma_w(c) = \frac{-(1 - \Theta'(c))G_i''(c)}{F_i''(c)G_i'(c) - F_i'(c)G_i''(c)} \quad (55)$$

$$\omega_w(c) = \frac{(1 - \Theta'(c))F_i''(c)}{F_i''(c)G_i'(c) - F_i'(c)G_i''(c)} \quad (56)$$

A.4 - Proof of Proposition 4

The proof of Proposition 4 follows from that of Proposition 2 and 3. As ODE (16) and (31), and the boundary conditions (19)-(20) and (34)-(35) are identical, the functions Θ , γ_u and ω_u are given from the results in Proposition 2 as

$$\gamma_u(c) = \frac{(V_1(c - I) - \Theta(c))G(c) - (V_1'(c - I) - \Theta'(c))G_0'(c)}{F_0'(c)G_0(c) - F_0(c)G_0'(c)}, \quad (57)$$

$$\omega_u(c) = \frac{(V_1(c - I) - \Theta(c))F(c) - (V_1'(c - I) - \Theta'(c))F_0'(c)}{F_0'(c)G_0(c) - F_0(c)G_0'(c)}. \quad (58)$$

and

$$\Theta(c) = \frac{\Phi + \lambda}{\Phi + \lambda + r} \left(\frac{\mu_0 + (\Phi + \lambda + r)c}{\Phi + \lambda + \delta} \right) + \frac{\lambda V_p(c)}{\Phi + \lambda + r} + \frac{\Phi}{\Phi + \lambda + r} (V_1(C^*) - C_1^* - I) \quad (59)$$

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Benchmark Parameters		
Expected Earnings (Before Investment)	μ_0	0.05
Expected Earnings (After Investment)	μ_1	0.10
Earnings Volatility	σ	0.10
Risk-free Rate	r	0.06
Cost-of-Carry	δ	0.02
Investment Cost	I	0.75
Recovery Value	α	0.00
Arrival Rate of Investors	Φ	1.50
Time-to-Finance (in years)	$1/\Phi$	0.66
Probability of Preemption	λ	0.50

Table 1: Benchmark Parameter Values.

Benchmark Optimal Thresholds (With Competition, $\lambda = 0.5$)		
Payout Threshold (After Investment)	C_1^*	0.3100
Payout Threshold (Before Investment)	C_W^*	0.3168
Investment Threshold	C_U^*, C_H^*	0.8544
Strategy Switching Threshold	C_L^*	0.5471
Benchmark Optimal Thresholds (No Competition, $\lambda = 0$)		
Payout Threshold (Before Investment)	C_W^*	0.3472
Investment Threshold	C_U^*, C_H^*	0.9544
Strategy Switching Threshold	C_L^*	0.9117

Table 2: Benchmark Parameter Values and Thresholds.

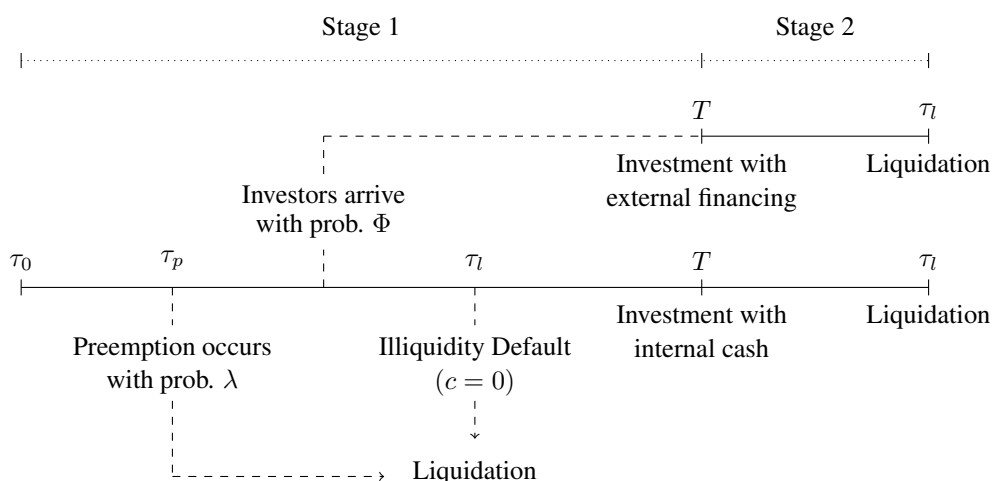


Figure 1: Timeline. The figure presents the time line and different stopping times of the firm. In stage 1, before investment occurs at time T , the firms risks being liquidated due to preemption with probability λ or due to illiquidity. We denote the time of default due to illiquidity as τ_l and the time of preemption as τ_p . The firm may generate sufficient internal funds or meet investors with probability Φ to enable investment. Once the firms invests it enters stage 2 and continues operations until liquidation occurs at time τ_l . The dashed line indicates that preemption, arrival of investors and illiquidity default may or may not occur over the time interval τ_0 to T .

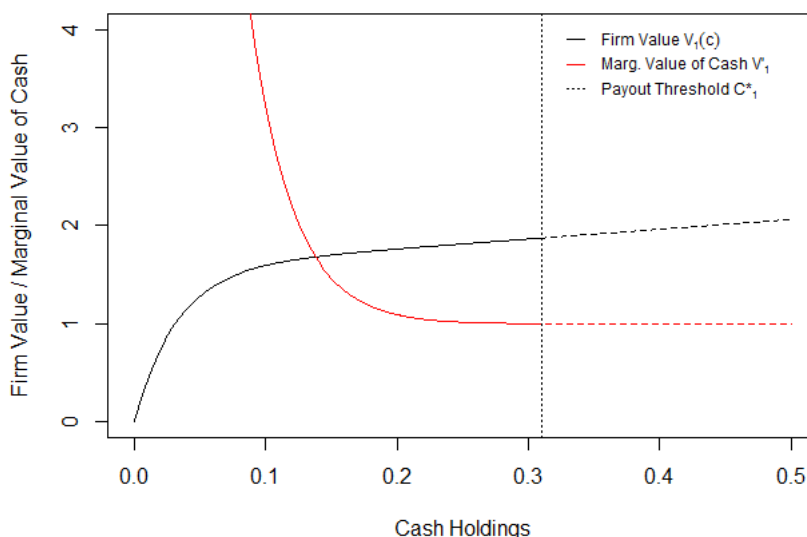


Figure 2: Firm Value, Marginal Value of Cash & Payout Boundary. The figure illustrates the value of the firm (the black line) and the marginal value of cash (the red line) after investment as a function cash. The vertical line represents the optimal payout boundary C_1^* . Below C_1^* it is optimal for the firm to retain earnings and search for investors, and above C_1^* the firms pays out all excess cash as dividends and the marginal value of cash is equal to one.

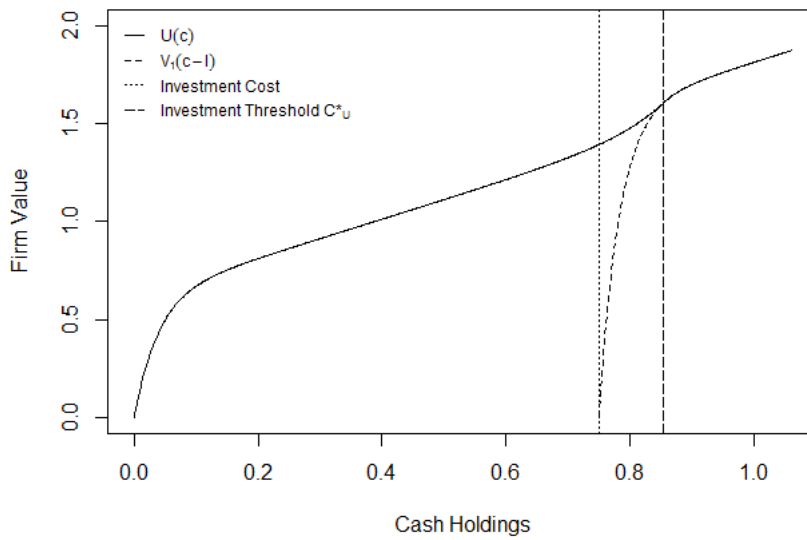


Figure 3: Firm Value & Investment Boundary. The figure illustrates the value of the firm as a function of cash. The solid line represent the value of the firm with a growth option, and the dashed line is the value of the firm after investment which merge with the value of the firm with a growth option at the optimal investment threshold C_U^* .

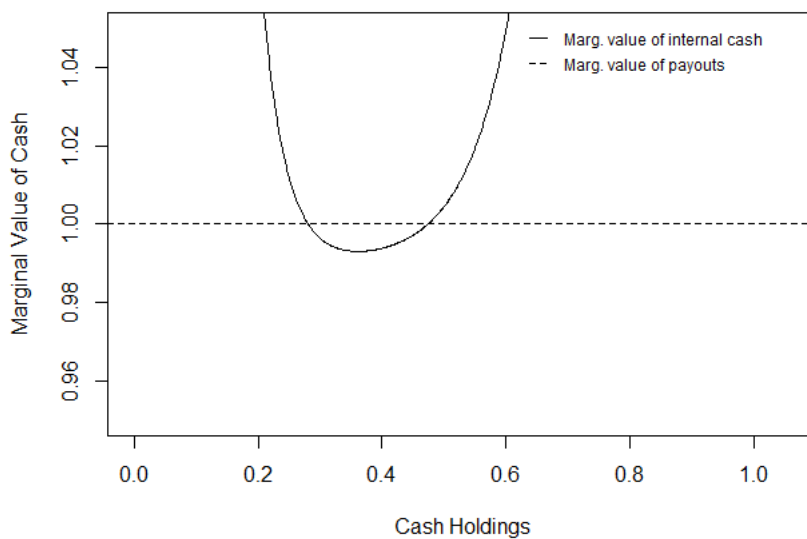


Figure 4: Marginal Value of Cash. The figure depicts the marginal value of cash (the solid line) as a function of cash for a firm relying on both internal cash and external financing. When the marginal value of cash goes below one, it will be optimal for the firm to pay out earnings as dividends rather than retain them within the firm. Hence, the firm should abandon the strategy of retaining cash and rely only on external financing.

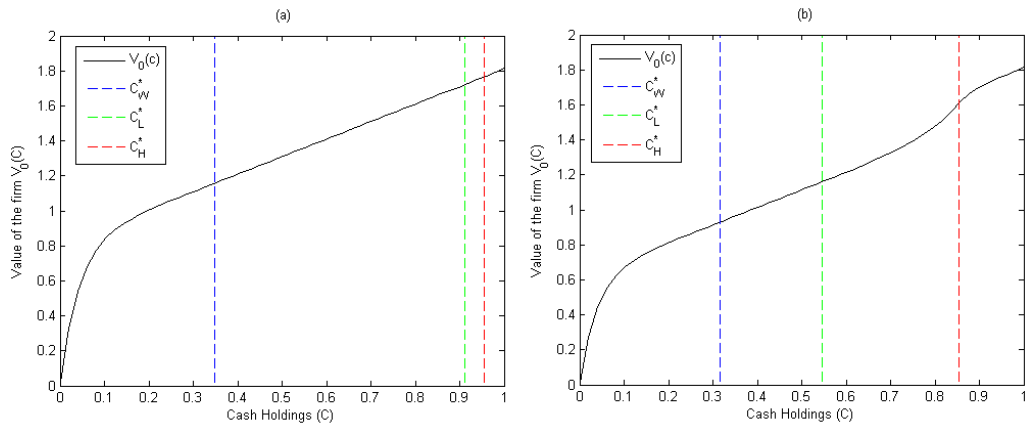


Figure 5: Optimal Switching Point & Firm Value. This figure illustrates the value of the firms as a function of cash. Panel (a) represents the case of no competition $\lambda = 0$, and panel (b) illustrates the case of $\lambda = 0.5$. In the area below C_W^* (the blue line) the optimal policy is to retain earnings and search for investors. In the area between C_W^* and C_L^* (the green line) it is optimal for the firm to pay a lump-sum dividend to reduce the cash holdings to C_W^* . Between C_L^* and C_H^* (the red curve) the optimal policy is to retain earnings and to search for investors, and in the area above C_H^* the firm's optimal policy is to invest with interval funds

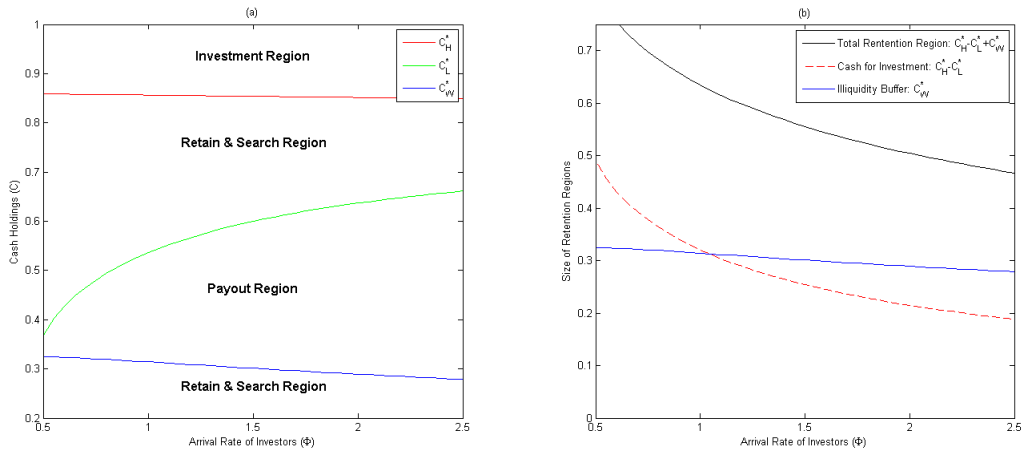


Figure 6: Optimal Switching Points & Time-to-Finance. Panel (a) illustrates the optimal investment, payout and switching thresholds as a function of the arrival rate of investors Φ . In the area below C_W^* (the blue line) the optimal policy is to retain earnings and search for investors. In the area between C_W^* and C_L^* (the green line) it is optimal for the firm to pay a lump-sum dividend to reduce the cash holdings to C_W^* . Between C_L^* and C_H^* (the red curve) the optimal policy is to retain earnings and search for investors, and in the area above C_H^* the firm's optimal policy is to invest with interval funds. Panel (b) illustrates the size of the retention and search regions measured as the difference between the upper and lower thresholds of the respective regions. The area between C_H^* and C_L^* can be interpreted as the cash held for investment purposes, whereas the area below C_W^* can be interpreted as the cash held for hedging illiquidity. Panel (b) illustrates the size of the retention regions. From this we see that both the upper, lower and total retention region is decreasing with the probability of finding an investors.

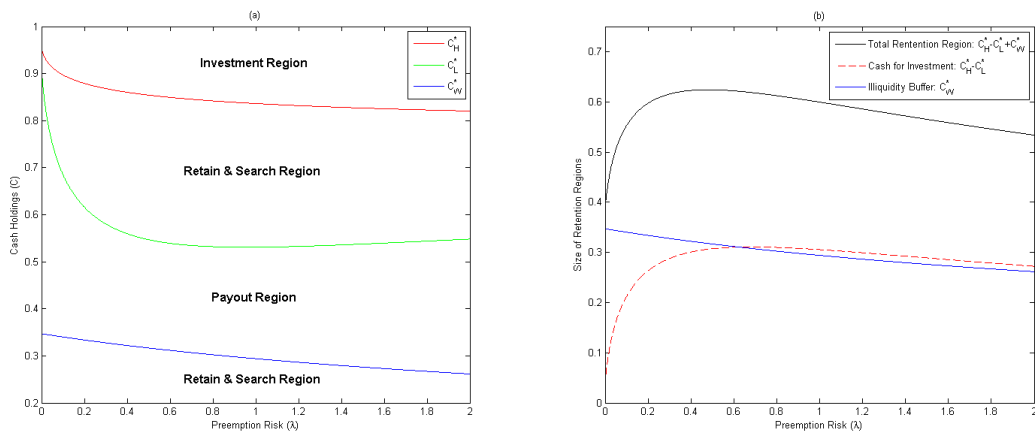


Figure 7: Optimal Switching Points & Preemption Risk. Panel (a) illustrates the optimal investment, payout and switching thresholds as a function of preemption risk. In the area below C_W^* (the blue line) the optimal policy is to retain earnings and search for investors. In the area between C_W^* and C_L^* (the green line) it is optimal for the firm to pay a lump-sum dividend to reduce the cash holdings to C_W^* . Between C_L^* and C_H^* (the red curve) the optimal policy is to retain earnings and search for investors, and in the area above C_H^* the firm's optimal policy is to invest with interval funds. Panel (b) illustrates the size of the retention and search regions measured as the difference between the upper and lower thresholds of the respective regions. The area between C_H^* and C_L^* can be interpreted as the cash held for investment purposes, whereas the area below C_W^* can be interpreted as the cash held for hedging illiquidity. From panel (b) we see that the lower retention region is decreasing in preemption risk, and that the upper region and that the total cash retention region is hump-shaped.

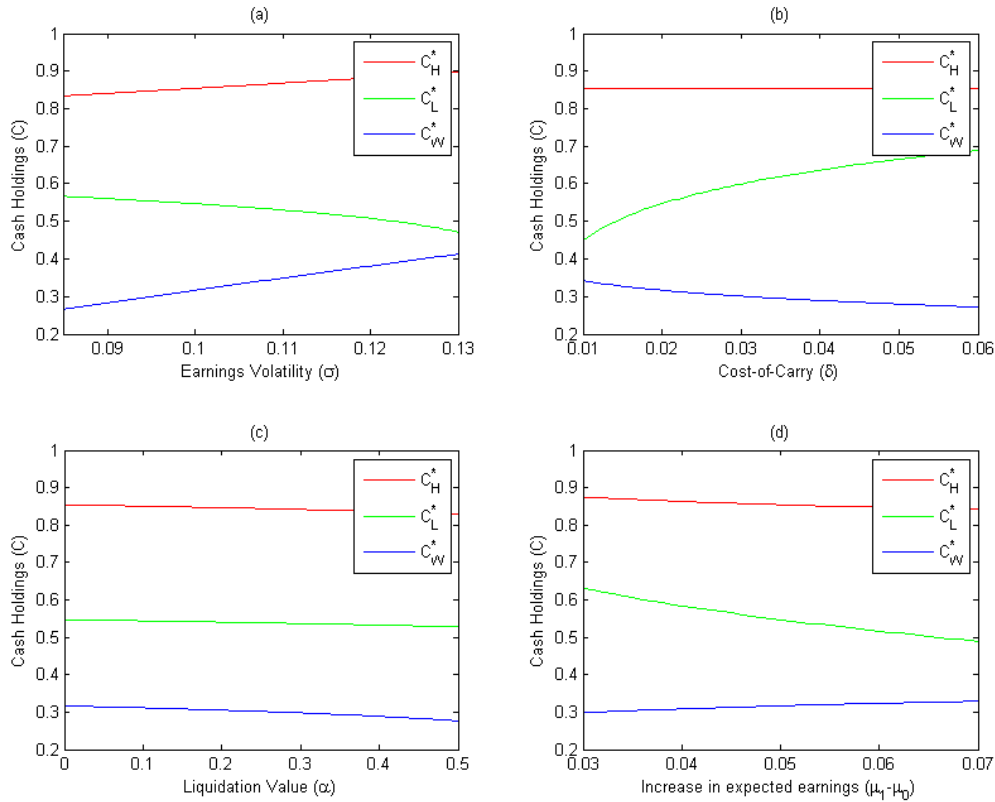


Figure 8: Optimal Switching Points. The figure depicts the optimal switching thresholds as a function of earnings volatility, cost-of-carry, liquidation value and profitability of the investment opportunity. The red line represents the optimal investment threshold C_H^* , the green line represents the optimal threshold C_L^* at which the firm switches between relying on both internal and external financing to only rely on external financing. The blue line represents the optimal payout threshold C_W^* of a firm only relying on external financing. The area between C_H^* and C_L^* illustrates the amount of cash held for investment purposes, whereas the area between C_W^* can be interpreted as the cash held for hedging illiquidity risk.